OCR RECOGNISING ACHIEVEMENT

Oxford, Cambridge and RSA Examinations

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2601

Pure Mathematics 1

Thursday **11 JANUARY 2001** Morning 1 hour 20 minutes

Additional materials:
- Answer booklet
- Graph paper
- MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.

Answer **all** questions.

You are permitted to use only a scientific calculator for this paper.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.

Final answers should be given to a degree of accuracy appropriate to the context.

The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

Section A (30 marks)

1 Expand $(1-2x)^3$, simplifying the terms. [2]

2 Find the gradient of the curve with equation $y = 2x^2 - 5x + 3$ at the point (3, 6). [3]

3 Sketch the graph of the function $y = |2x-1|$, and state the coordinates of any points where the graph meets the axes. [3]

4 Find the root of the equation $\sin x = 2\cos x$ for which $90° < x < 270°$. Give your answer to the nearest 0.1°. [3]

5

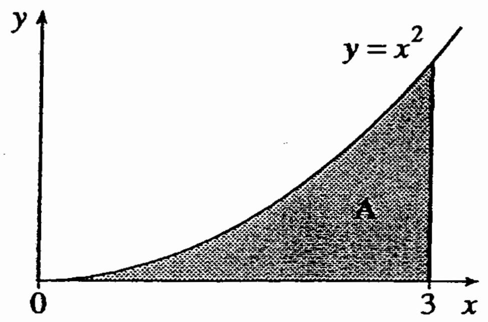

Fig. 5

Fig. 5 shows the region A bounded by the curve with equation $y = x^2$, the x-axis and the line $x = 3$. Find the volume of revolution generated when A is rotated through 360° about the x-axis. [3]

6 Given that $x - 1$ is a factor of the polynomial $x^3 + kx^2 + 6x - 5$, find the value of the constant k.

Hence express the polynomial as a product of $x - 1$ and a quadratic factor. [4]

7 The gradient of a curve is given by

$$\frac{dy}{dx} = 6x^2 - 5x.$$

The curve passes through the point (–1, 3). Find the equation of the curve. [4]

8 Fig. 8 shows the shaded sector PCQ of a circle of radius 4 cm. The area of the sector is 36 cm^2.

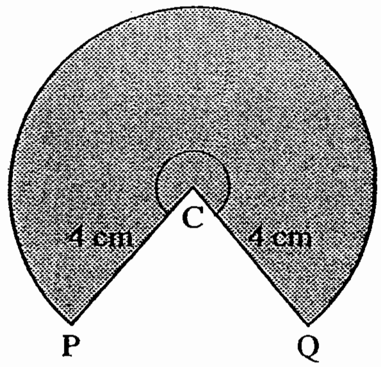

Fig. 8

Find, in radians, the size of the reflex angle PCQ. Find also the perimeter of the shaded sector PCQ. [4]

9 Fig. 9 illustrates a triangle ABC in which angle A is exactly 30°.

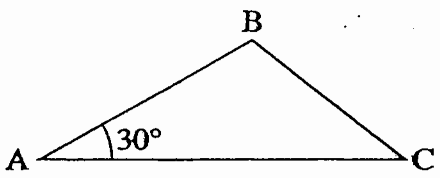

Fig. 9

AB is measured as 8 cm to the nearest centimetre and AC is measured as 12 cm to the nearest centimetre.

Find the area of the triangle using these data. Find also the largest possible absolute error in your answer. [4]

Section B (30 marks)

10 (i) Fig. 10.1 shows the rate of flow of water, in litres per minute, into a barrel during a sudden downpour of rain.

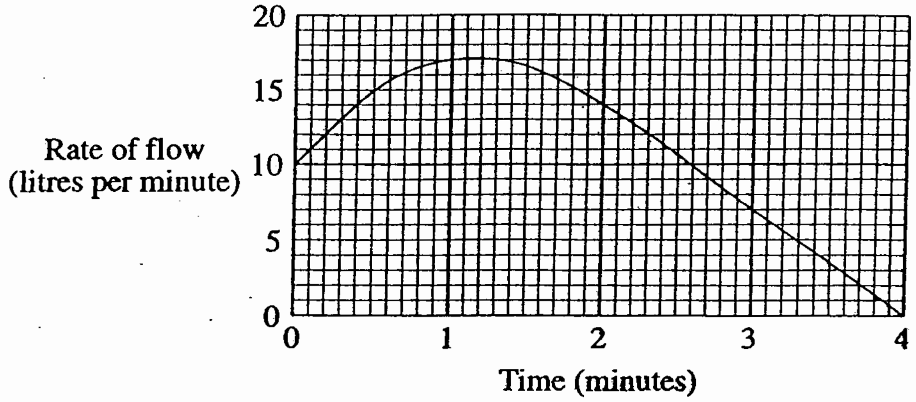

Fig. 10.1

Use the trapezium rule with 4 strips to estimate the area of the region between the curve and the time axis for the period 0 to 4 minutes. State what information is given by this area. [4]

(ii) Fig. 10.2 shows the rate of flow, r litres per minute, into the barrel on a second occasion.

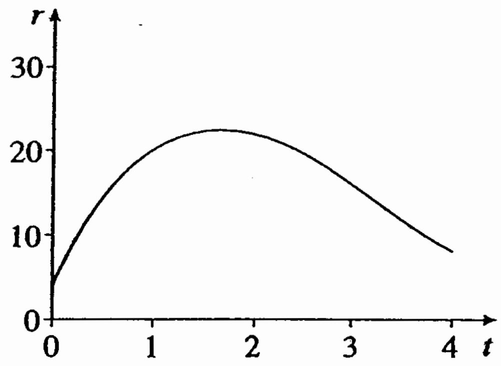

Fig. 10.2

The equation connecting r and t is $r = t^3 - 10t^2 + 25t + 4$ for $0 \leqslant t \leqslant 4$, where t is the time in minutes.

(*A*) Find the exact value of the area between the curve and the t-axis for the period $t = 0$ to $t = 4$. [3]

(*B*) Find the exact value of t for which the rate of flow is a maximum. [4]

Question 10 continues on page 5

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The volume of water in the barrel at time t minutes is V litres.

(iii) Fig. 10.3 shows the graph of V against t on a third occasion.

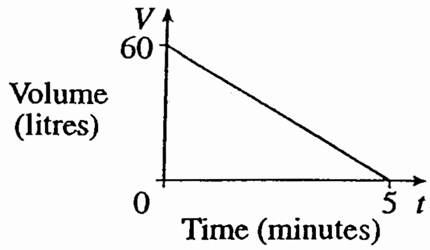

Fig. 10.3

Describe the rate of flow shown by this graph and state the equation connecting V and t for $0 \leqslant t \leqslant 5$. [2]

(iv) On a fourth occasion, the barrel initially contains 40 litres of water and is emptied in 6 minutes, with the water flowing out at a decreasing rate.

Sketch a possible graph of V against t. [2]

11 **(i)** Show that the equation $x^2 - 6x + y^2 - 2y = 6$ represents a circle with centre (3, 1). Find the radius of this circle. [3]

(ii) Find the coordinates of the points P and Q (as in Fig. 11) where this circle meets the x-axis. [3]

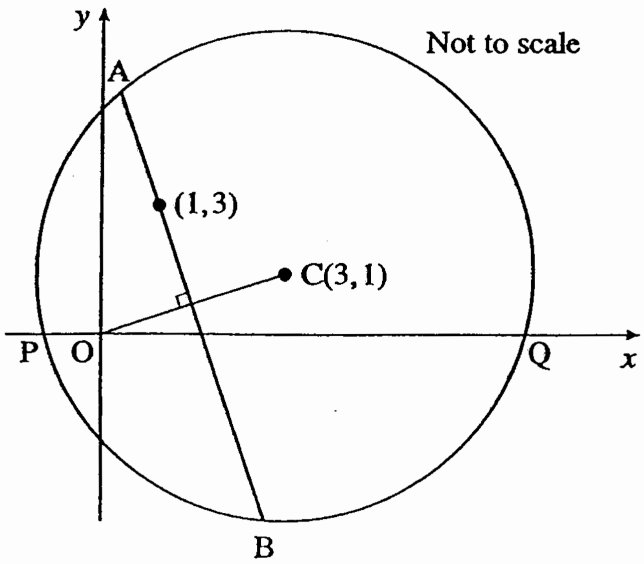

Fig. 11

Fig. 11 shows the circle with equation $x^2 - 6x + y^2 - 2y = 6$ and its centre C. The chord AB passes through the point (1, 3) and is perpendicular to OC, where O is the origin.

(iii) Find the coordinates of the points A and B.

Determine whether AB is longer or shorter than PQ. [9]

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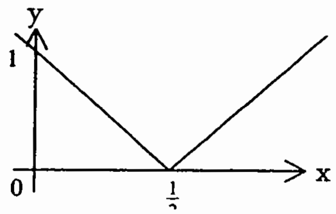
Mark Scheme

SECTION A

1.	$1 - 6x + 12x^2 - 8x^3$	B2	sc 1 for 1, 6, 12, 8 or 1, 6, 6, 2 (ignoring signs), or for 3 terms correct	2
2.	$4x - 5$ 7	B1 B1 B1 ft	−1 extra term(s) ft their $y' = ax + b$	3
3.	Line segment for $y = 2x - 1$ Line segment for $y = 1 - 2x$ [for above marks allow lines continuing beyond correct segs.] Fully correct and labelled intersections	B1 B1 B1		3
4.	243.4°	B3	M1 for $\tan x = 2$ oe eg $\sin x = \frac{2}{\sqrt{5}}$ A1 or B2 63.4	3
5.	$\int_0^3 \pi x^4 dx$ $\left[\frac{\pi x^5}{5}\right]_0^3$ $243\pi/5$ oe or 153 or 152.6… or 152.7	M1 M1 dep A1	Condone without limits SC1 for $\left[\frac{\pi x^3}{3}\right]_0^3$ after $\int_0^3 \pi x^2 dx$ Or for $\left[\frac{x^5}{5}\right]_0^3$ after $\int_0^3 x^4 dx$	3
6.	$1 + k + 6 - 5 = 0$ $k = -2$ $x^2 - x + 5$	M1 A1 or B2 B2	Subst of $x = 1$, or long divn, to obtain eqn in k B1 for 2 terms correct	4
7.	$[y =]\ 2x^3 - 5x^2/2\ [+ c]$ o.e. Subst of $y = 3$, $x = -1$ $c = \frac{15}{2}$ or 7.5	B1+B1 M1 A1	Condone omission of c May only be earned if c included or B4 for $[y =]2x^3 - \frac{5x^2}{2} + \frac{15}{2}$ or equivalent	4
8.	$\frac{1}{2} \times 4^2 \times \theta = 36$ $\theta = 4.5$ isw $2r + r\theta$ used with $r = 4$ perimeter = 26	M1 A1 or B2 M1 A1 or B2	Tolerance ± 0.01 SC1 for 18 Tolerance ± 0.05	4

9.	24 (cm^2) cao upper bounds 8.5 and 12.5 error = |their 24 – their ½ × 8.5 × 12.5 × sin 30°| = 2.5625 as final answer	B1 B1 M1 A1 or B2	Condone use of lower bounds or with wrong upper bounds or with angles 29-31° Allow 2.56, 2.6 or more s.f.	4
			Total Section A	30

SECTION B

10	(i)	½ × 1 × (27 + 31 + 21 + 7) or ½ × (10 + 0 + 2 [17 + 14]) = 43	M2 A1	M1 for partially correct application of trap rule Condone wrong/omitted units	
		volume of water [put into barrel] of litres of water or rain, etc	1	condone 'volume of water in barrel'. 0 for 'litres' alone4	4
	(ii)	(A) $\left[\frac{t^4}{4} - \frac{10t^3}{3} + \frac{25t^2}{2} + 4t\right]_0^4$	M2	M1 for 3 terms correct M0 for trapezium rule	
		$= 66\frac{2}{3}$	A1	Condone 66.7 or 66.66...	3
		(B) $dr/dt = 3t^2 - 20t + 25$	M1		
		$dr/dt = 0$	M1		
		$(3t - 5)(t - 5) = 0$ or correct use of formula	A1		
		at max $t = \frac{5}{3}$ or exact equiv	A1	A0 for 1.67	4
	(iii)	constant flow [out] $V = 60 - 12t$	1 1	Allow 'steady decrease in volume'	2
	(iv)	sketch satisfying: through (0, 40) and (6, 0) gradient getting less steep	 1 1	 Condone some neg V	 2
11	(i)	$(x - a)^2 + (y - b)^2 = r^2$ quoted and used $(x - 3)^2 + (y - 1)^2 = 6 + 9 + 1$ or 16 $r = 4$	M1 M1 B1	May start from equation given or equation of circle or $r^2 - 10 = 6$	3
	(ii)	$(x - 3)^2$ $= 15$ (their $r^2 - 1$) or $x^2 - 6x - 6 = 0$ $x = 3 \pm \sqrt{15}$ or 6.87 and – 0.87 to 1 or more dp	 M1 A1 + A1	For subst $y = 0$ in their circle eqn and rearranging 2 = 0 or their $r^2 - 1$ or in eqn given, rearranging that to = 0 Allow seen in these formats as well as coords given as asked; B3 for both sols	3
	(iii)	grad AB = –3 $y - 3 = -3(x - 1)$ $y = -3x + 6$	M1 M1 A1 or B3	Or $y = -3x + C$ and (1, 3) subst; ft their gradient o.e. simplified version	
		subst their equation for AB in equation for circle	M1		
		simplify to $ax^2 + bx + c = 0$ $(x - 3)(5x - 3) = 0$ (3, –3) or (0.6, 4.2)	M1 A1 A1 or B4	or $ay^2 + by + c = 0$ SC B4 for (7, 1) or (0.2, 3.4) following grad OC = 3	
		AB = $\sqrt{(2.4^2 + 7.2^2)}$ or $\sqrt{57.6}$ or 7.89… to 1 dp or more	 1		
		PQ = $2\sqrt{15}$ or $\sqrt{60}$ or 7.75 or 7.7 [so PQ longer]	 1	Allow 7.74(…) or $\sqrt{59.9}$...	9
				Total Section B	30
				Total for paper	60

Examiner’s Report

Report of the Examiners on MEI Structured Mathematics

January 2001

Pure Mathematics 1 (2601)

General Comments

The marks gained covered the whole range from 0 to 60. There was a good number of high-scoring scripts where candidates were extremely well-prepared and tackled the paper with confidence. Regretfully, there was also a large number of candidates who gained less than 10 marks. Some low-scoring candidates had minimal skills at this level, whilst other low achievers gave the impression that they had been entered for this module too early – they had a small number of quality answers being balanced by several omissions, presumably of topics not yet met. Centres are clearly experimenting with entry policies, but it was sad that for some candidates that this first experience of AS mathematics had been so negative. Another factor was that some candidates seemed unaware that fewer formulae are provided in the booklet than for the former syllabus – lack of knowledge of basic formulae reduced their achievement.

Those candidates who used long methods, as in question 9, sometimes ran out of time in question 11. However, candidates of all abilities tackled the long questions sensibly, using their in-built accessibility to attempt just some parts of the questions – sometimes the later parts.

Comments on individual questions

Question 1 (Binomial expansion)

What was intended as a straightforward start to the paper was well done by many of those familiar with the binomial expansion, although errors with the coefficients were also frequent. Others attempted multiplication, but made many algebraic and numerical errors in doing so.

$1-6x+12x^2-8x^3$.

Question 2 (Gradient)

Most candidates differentiated correctly, although a few failed to substitute correctly to gain the required answer.

7.

Question 3 (Sketch graph)

As in previous P1 papers, familiarity with modulus was limited. Many did not realise that $y \geq 0$. Many candidates gained only one mark, for a correct sketch of $y = 2x - 1$, and did not attempt to draw the other part of the graph.

(0.5,0) and (0,1).

Question 4 (Trigonometrical equation)

This question tended to be either correct or not attempted. Those who appreciated that $\tan x = 2$ were, as expected, usually more successful than those who attempted use of $\sin^2 x + \cos^2 x = 1$.

243.4°.

Question 5 (Area and volume of a graph)

Volume of revolution was an unfamiliar topic to some candidates. Some could not cope with the π factor when integrating, whilst others attempted to integrate $\int x^2 \mathrm{d}x$ etc.

$\frac{248\pi}{5}$.

Question 6 (Factor theorem)

Those who knew and used the factor theorem fared better than those who attempted long multiplication, though some correct solutions were seen using such methods.

$k = -2$; $x^2 - x + 5$.

Question 7 (Integration)

Examiners were pleased that those who integrated frequently included the constant of integration, although some errors in substitution were then made by some candidates. Many candidates, however, found the equation of the tangent to the curve at the given point, rather than the equation of the curve.

$y = 2x^3 - \frac{5x^2}{2} + \frac{15}{2}$.

Question 8 (Area and perimeter of a sector)

Many misquotes of the radian formulae for arc length and sector area were seen. Many candidates worked in degrees and then tried to covert back to radians. Those who did obtain the correct answer sometimes went on to introduce a factor of π before going on. Some good candidates who gained 59 marks lost one mark in this question, as did others, for finding the arc length of the sector rather than its perimeter.

4.5; 26 cm.

Question 9 (Area of a triangle)

Many did not know or use the area formula $A = \frac{1}{2}ab\sin C$, making this question much more difficult for themselves. Some used very long methods for finding the height of the triangle. The correct upper bounds were often used, but some candidates subtracted lower bound answers to find the error, or found relative instead of absolute error.

24 cm^2; 2.5625 cm^2.

Question 10 (Modelling rate of flow)

(i) This was usually well-answered, although some misapplied the trapezium rule. The formula for this continues to be given in the booklet.

(ii) A few continued to apply the trapezium rule in part (*A*), but many integrated correctly, with any errors usually concerning integration of the constant term. In part (*B*), most differentiated correctly and equated their result to zero and were able to factorise or use the formula to obtain the correct answer.

(iii) Some students confused volume and flow rate, stating that the rate of flow was decreasing rather than constant. A good number were able to state the equation connecting V and t.

(iv) A common error was to draw a straight line graph connecting the relevant points on the axes, but many good graphs were seen.

(i) 43 litres, the total rainfall collected in the four minutes; (ii) (A) $66\frac{2}{3}$, (B) $1\frac{2}{3}$;
(iii) constant flow out, $V = 60 - 12t$; (iv) sketch graph.

Question 11 (Circles and coordinate geometry)

(i) There were excellent demonstrations from many candidates, but also confused algebraic manipulation from others, particularly from those who were not sure of the form of a circle equation.

(ii) Many were able to obtain the coordinates of P and Q although there were some errors in using the quadratic formula and some candidates substituted $x = 0$ instead of $y = 0$. Some successfully found P and Q by using Pythagoras' theorem.

(iii) Most students were able to find the equation of AB, although some wasted time by first finding the equation of OC. The examiners were pleased that many who got this far went on to use the correct method for finding the intersection of AB with the circle, although errors in manipulation of the algebra were frequent. Some candidates used alternative methods to establish which of PQ and AB was longer, such as using the fact that the longer chord is a shorter distance from the centre of the circle.

(i) e.g. showing that $(x-3)^2 + (y-1)^2 = 16$, 4; (ii) (6.87,0) and (–0.87,0);
(iii) (3, –3) and (0.6, 4.2), $PQ = \sqrt{60}$, $AB = \sqrt{57.6}$ so PQ is longer.